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DYNAMICS OF THIN FILMS IN A MAGNETIC FIELD

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The oscillations of thin conducting films placed in a magnetic field are considered. The effect of the field in different directions on the effective elasticity of the film is described and dispersion relations are obtained for longitudinal and transverse waves.

It is well known from the theory of elasticity [1] that the properties of strain waves in an isotropic medium are different from those of waves in thin films. For example, waves which are normal to the plane of the film exhibit dispersion and the phase velocity of longitudinal waves is altered. We might expect that new deformation modes will occur in a conducting film placed in a magnetic field and that the properties of these will differ from those of magnetic-field transport waves owing to the deformations in the three-dimensional elastic conducting medium.

Effects related to the presence of an external magnetic field should begin to appear at much smaller field values because the characteristic velocity in a magnetic field increases as the thickness of the film is reduced. For thin conducting films it is possible by proper choice of the parameters to get the magnetoelastic velocity greater than the velocity of sound; i.e., the nature of the strain propagation in the film will be determined mainly by the magnetic field.

We shall consider the propagation of deformations in a thin perfectly conducting film of thickness d placed in an external homogeneous constant magnetic field **H**. In order to get the equation for the deformation **u**, we make use of the equilibrium equations for a thin elastic film [1]. We introduce a Cartesian system of coordinates x, y, z so that the film lies in the (x, y) plane and the external magnetic field is in the (x, z) plane at an angle α to the x axis. For the displacement u_z we have

$$[Ed^3/12(1-\tau^2)]\,\Delta^2 u_z - P_z = 0. \tag{1}$$

For the displacements u_x and u_y we have

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$$Ed\left[\frac{1}{1-\tau^2}\frac{\partial^2 u_x}{\partial x^2}+\frac{1}{2(1+\tau)}\frac{\partial^2 u_x}{\partial y^2}+\frac{1}{2(1-\tau)}\frac{\partial^2 u_y}{\partial x\partial y}\right]+P_x=0; \quad (2)$$

$$Ed\left[\frac{1}{1-\tau^2}\frac{\partial^2 u_y}{\partial y^2}+\frac{1}{2(1+\tau)}\frac{\partial^2 u_y}{\partial x^2}+\frac{1}{2(1-\tau)}\frac{\partial^2 u_x}{\partial x\partial y}\right]+P_y=0, \quad (3)$$

where E is Young's modulus, τ is Poisson's ratio, d is the thickness of the film, and **P** is the surface density of the force which is producing the deformation. The equations for the oscillations of a conducting film in a magnetic field can be obtained from (1)-(3) if we replace **P** by

$$\mathbf{P}=-\rho d\frac{\partial^2\mathbf{u}}{\partial t^2}+\frac{1}{c}[\mathbf{i}\times\mathbf{H}], \quad (4)$$

where ρ is the volume density of the film, c is the velocity of light, and **i** is the surface current density in the film produced by the displacement in the external magnetic field. The first term in (4) is the product of the acceleration $d^2\mathbf{u}/dt^2$ and the mass ρd of unit surface of the film, and the second term is the surface density of the force acting on the film as the surface current **i** flows in the external magnetic field **H**. Substituting (4) into (1)-(3), we get

$$\frac{\partial^2 u_x}{\partial t^2}-c_l^2\frac{\partial^2 u_x}{\partial x^2}-c_t^2\frac{\partial^2 u_x}{\partial y^2}-\left(c_l^2-c_t^2\right)\frac{\partial^2 u_y}{\partial x\partial y}-\frac{1}{\rho cd}[\mathbf{i}\times\mathbf{H}]_x=0; \quad (5)$$

$$\frac{\partial^2 u_y}{\partial t^2}-c_l^2\frac{\partial^2 u_y}{\partial y^2}-c_t^2\frac{\partial^2 u_y}{\partial x^2}-\left(c_l^2-c_t^2\right)\frac{\partial^2 u_x}{\partial x\partial y}-\frac{1}{\rho cd}[\mathbf{i}\times\mathbf{H}]_y=0; \quad (6)$$

$$\frac{\partial^2 u_z}{\partial t^2}+c_\perp^2\frac{d^2}{4}\Delta^2 u_z-\frac{1}{\rho cd}[\mathbf{i}\times\mathbf{H}]_z=0, \quad (7)$$

where

$$c_l=\sqrt{E/\rho(1-\tau^2)};\; c_t=\sqrt{E/2\rho(1+\tau)};\; c_\perp=\sqrt{E/3\rho(1-\tau^2)}.$$

In order to close the system (5)-(7) we have to express the current **i** in terms of the deformation **u**. We therefore consider the perturbation in the magnetic field **h** and the electric field **E** produced in the whole of space by the oscillations of the conducting film in the external field.* The fields are determined from the Maxwell equations in the regions $z>0$ and $z<0$:

$$\operatorname{rot}\mathbf{E}=-(1/c)\partial\mathbf{h}/\partial t;\; \operatorname{rot}\mathbf{h}=(1/c)\partial\mathbf{E}/\partial t;\; \operatorname{div}\mathbf{h}=0. \quad (8)$$

In the plane $z=0$ the field equations transform into the boundary conditions [2]

$$h_y(+0)-h_y(-0)=-(4\pi/c)i_x;$$
$$h_x(+0)-h_x(-0)=(4\pi/c)i_y;\; i_z=0. \quad (9)$$

In addition, the electric field in the plane $z=0$ can be expressed for a perfectly conducting film in terms of the deformations as

$$\mathbf{E}_0(x,y,0)=-(1/c)\partial[\mathbf{u}(x,y)\times\mathbf{H}]/\partial t. \quad (10)$$

Under the boundary condition (10) the electric field can be found by means of (8) and thus the magnetic field can be determined above and below the plane $z=0$. The discontinuity of the magnetic field in the plane $z=0$ is related through (9) with the quantities **i** and **u**.

We assume that all quantities depend on the time t and the radius vector **r** in the plane (x, y) as $\exp\{-i\omega t+i\mathbf{kr}\}$ so that the field satisfies the equation

$$d^2\mathbf{E}/dz^2-q^2\mathbf{E}=0;\; q^2=k^2-\omega^2/c^2;\; \operatorname{Re}q>0. \quad (11)$$

The solution of (11) under the boundary conditions (10) is

$$\mathbf{E}(k,\omega,z)=\mathbf{E}_0(k,\omega)\exp(-qz),\; z>0;$$
$$\mathbf{E}(k,\omega,z)=\mathbf{E}_0(k,\omega)\exp(qz),\; z<0. \quad (12)$$

We can get the expression for the tangential components of **h** from (8) by using (12):

$$h_x=(c/i\omega)\{ik_yE_{0z}\exp(-qz)+qE_{0y}\exp(-qz)\},$$
$$h_y=(c/i\omega)\{-qE_{0x}\exp(-qz)-ik_xE_{0z}\exp(-qz),\; z>0;$$
$$h_x=(c/i\omega)\{ik_yE_{0z}\exp(qz)-qE_{0y}\exp(qz)\},$$
$$h_y=(c/i\omega)\{qE_{0x}\exp(qz)-ik_xE_{0z}\exp(qz)\},\; z<0. \quad (13)$$

*Equations (1)-(3) are linear provided that the film parameters are independent of the displacement, i.e., that the condition $u\ll\lambda$ is satisfied. Under this condition the relationship (4) does not destroy the linearity of Eqs. (1)-(3) if the magnitudes of the perturbations **h** and **E** satisfy the inequalities $\mathbf{h}\ll\rho d\lambda\omega^2/H$, $\mathbf{E}\ll\omega\lambda H/c$.

Substituting (13) into (9) and using (10) we get

$$\mathbf{i}(k,\omega) = (c/2\pi)k[\mathbf{u}(k,\omega)\times\mathbf{H}]. \tag{14}$$

In (14) we neglect effects associated with the finite velocity of light; i.e., we put $q \approx k$. Expression (14) closes the system (5)-(7), which can now be written in the form

$$\begin{gathered}
\left\{-\omega^2 + c_l^2k_x^2 + c_t^2k_y^2 + 2a^2\frac{k}{d}\sin^2\alpha\right\}u_x + \left(c_l^2 - c_t^2\right)k_xk_yu_y - 2a^2\frac{k}{d}\sin\alpha\cdot\cos\alpha\, u_z = 0;\\
\left(c_l^2 - c_t^2\right)k_xk_yu_x + \left\{-\omega^2 + c_l^2k_y^2 + c_t^2k_x^2 + 2a^2\frac{k}{d}\right\}u_y = 0;\\
-2a^2\frac{k}{d}\sin\alpha\cdot\cos\alpha\cdot u_x + \left\{-\omega^2 + c_\perp^2\frac{k^4}{4} + 2a^2\frac{k}{d}\cos^2\alpha\right\}u_z = 0,
\end{gathered} \tag{15}$$

where $a^2 = H^2/4\pi\rho$.

We consider the case $\alpha = \pi/2$. It follows from (15) that the magnetic field has no effect here on the deformation u_z, and for the tangential deformation $\mathbf{u}_k$ we get (15)

$$-\omega^2\mathbf{u}_k + c_t^2k^2\mathbf{u}_k + \left(c_l^2 - c_t^2\right)\mathbf{k}\,(\mathbf{k}\cdot\mathbf{u}_k) + \frac{2a^2}{d}k\cdot\mathbf{u}_k = 0. \tag{16}$$

From (16) we can obtain the dispersion relations for longitudinal and transverse waves. In order to do this we represent $\mathbf{u}_k$ as

$$\mathbf{u}_k = \mathbf{u}_l + \mathbf{u}_t, \tag{17}$$

and $(\mathbf{k}\cdot\mathbf{u}_t) = 0$ since the transverse deformations do not change the volume and thus div $\mathbf{u}_t = 0$ and also $[\mathbf{k}\times\mathbf{u}_l] = 0$, since curl $\mathbf{u}_l = 0$. Substituting (17) into (16) and carrying out a scalar multiplication by k, we get

$$\mathbf{k}\cdot\left\{-\omega^2\mathbf{u}_l + c_l^2k^2\mathbf{u}_l + \frac{2a^2}{d}k\cdot\mathbf{u}_l\right\} = 0.$$

Since the vector product of k and the quantity in brackets is equal to zero, it follows that the expression in the brackets itself is equal to zero and we thus get the relationship for the longitudinal waves. A similar relationship is obtained for transverse waves with c_l replaced by c_t. Thus the equations for the two types of wave separate, and the dispersion relations can be written

$$\omega_{l,t} = \sqrt{c_{l,t}^2k^2 + \frac{2a^2}{d}k}. \tag{18}$$

It can be seen from (18) that when $a = 0$, c_l and c_t are the phase velocities of the longitudinal and transverse waves, respectively. The inclusion of a magnetic field produces dispersion and the waves propagate with group velocities $\mathbf{U} = \partial\omega/\partial\mathbf{k}$, which can be found from (18) to be

$$\mathbf{U}_{l,t} = \frac{c_{l,t}^2\varkappa + a^2}{\sqrt{c_{l,t}^2\varkappa^2 + 2a^2\varkappa}}\cdot\frac{\mathbf{k}}{k}, \qquad \varkappa = kd. \tag{19}$$

For strong magnetic fields $(a \gg c_{l,t}\sqrt{\varkappa})$ we get from (19) that $U_H = a/\sqrt{2\varkappa}$; i.e., the velocity increases as $\varkappa$ is reduced. This increase in velocity can be explained by the fact that as $\varkappa$ is reduced, the thickness of the film and hence the mass per unit area decreases, and for a fixed surface density of force this leads to an increase in the velocity of motion of unit surface area.

We now consider the case $\alpha = 0$. The system (15) becomes

$$\begin{gathered}
\{-\omega^2 + c_l^2k_x^2 + c_t^2k_y^2\}\,u_x + \left(c_l^2 - c_t^2\right)k_xk_yu_y = 0;\\
\left(c_l^2 - c_t^2\right)k_xk_yu_x + \left\{-\omega^2 + c_l^2k_y^2 + c_t^2k_x^2 + 2a^2\frac{k}{d}\right\}u_y = 0;\\
\left\{-\omega^2 + c_\perp^2\frac{k^4}{4} + 2a^2\frac{k}{d}\right\}u_z = 0.
\end{gathered} \tag{20}$$

It can be seen from (20) that the wave u_z propagates independently of $\mathbf{u}_k$ and its dispersion relation is

$$\omega = \sqrt{c_\perp^2/4\cdot k^4 + 2a^2/d\cdot k}.$$

The group velocity is given by the expression

$$\mathbf{U}_\perp = \frac{c_\perp^2\varkappa^3 + 2a^2}{\sqrt{c_\perp^2\varkappa^4 + 8a^2\varkappa}}\cdot\frac{\mathbf{k}}{k}. \tag{21}$$

For strong magnetic fields we get from (21)

$$\mathbf{U}_\perp = (a/\sqrt{2\varkappa})\cdot \boldsymbol{k}/k = \mathbf{U}_H.$$

According to (21), the quantity $U_\perp$ as a function of $\varkappa$ has a minimum at the point $\varkappa_0 = 0.8(a/c_\perp)^{2/3}$, and $|U(\varkappa_0)| = 0.92a^{2/3}c_\perp^{1/3}$. When $\varkappa \gg \varkappa_0$, $U \approx c_\perp \varkappa$. It has to be remembered that all these equations are valid for $\varkappa \ll 1$, since it is assumed that $d \ll \lambda$.

From (20) we have the dispersion relation for $\mathbf{U}_k$

$$\omega^4 - \omega^2\left[\left(c_l^2 + c_t^2\right)k^2 + \frac{2a^2}{d}k\right] + \left[c_l^2c_t^2k^4 + \left(c_l^2k_x^2 + c_t^2k_y^2\right)\frac{2a^2}{d}k\right] = 0,$$

whence

$$\omega^2 = \frac{1}{2}\left[\left(c_l^2 + c_t^2\right)k^2 + \frac{2a^2}{d}k\right] \pm \sqrt{\frac{1}{4}\left[\left(c_l^2 + c_t^2\right)k^2 + \frac{2a^2}{d}k\right]^2 - \left[c_l^2c_t^2k^4 + \left(c_l^2k_x^2 + c_t^2k_y^2\right)\frac{2a^2}{d}k\right]}.$$

For strong magnetic fields ($a \gg c_l\sqrt{\varkappa}$) the frequency depends on k in the following way:

$$\omega_1 = a\sqrt{\frac{2k}{d}} - \left(c_l^2k_x^2 + c_t^2k_y^2\right)\frac{1}{2a}\sqrt{\frac{d}{2k}}, \qquad \omega_2 = \sqrt{c_l^2k_x^2 + c_t^2k_y^2}.$$

The group velocity corresponding to the frequency ω_1 is

$$U_{1x} = \left\{\frac{a}{\sqrt{2kd}} - \frac{c_l^2}{a}\sqrt{\frac{kd}{2}} + \frac{c_l^2k_x^2 + c_t^2k_y^2}{4ak^2}\sqrt{\frac{kd}{2}}\right\}\frac{k_x}{k};$$

$$U_{1y} = \left\{\frac{a}{\sqrt{2kd}} - \frac{c_t^2}{a}\sqrt{\frac{kd}{2}} + \frac{c_l^2k_x^2 + c_t^2k_y^2}{4ak^2}\sqrt{\frac{kd}{2}}\right\}\frac{k_y}{k}.$$

The group velocity along the wave vector **k** depends on the angle φ = arctan k_y/k_x as

$$U_l/U_H = \left(\mathbf{U}_1\cdot\frac{\mathbf{k}}{k}\right)\Big/U_H = 1 - \frac{3}{4}\,\frac{c_l^2\cos^2\varphi + c_t^2\sin^2\varphi}{a^2}\,\varkappa, \qquad c_l > c_t.$$

We can find the dependence of the transverse group velocity U_t on the angle φ from the condition $U_t^2 = U^2 - U_l^2$. Substituting the values of U and U_l for U_t we get

$$U_t = \frac{c_l^2 - c_t^2}{a}\sin\varphi\cdot\cos\varphi\cdot\sqrt{\frac{\varkappa}{2}}.$$

Thus the group velocity coincides with the direction of propagation of the wave only when this propagation is along the magnetic field or transverse to it. For other angles φ the direction of $\mathbf{U}_1$ does not coincide with that of **k**.

It should be noted that in the present case there is an extra wave mode, which satisfies the dispersion relationship $\omega_2 = \sqrt{c_l^2k_x^2 + c_t^2k_y^2}$ and is independent of the magnetic field, but this only occurs in strong fields (in the sense in which this term was used above). The components of the group velocity are

$$U_{2x} = \frac{c_l^2\cos\varphi}{\sqrt{c_l^2\cos^2\varphi + c_t^2\sin^2\varphi}}; \quad U_{2y} = \frac{c_t^2\sin\varphi}{\sqrt{c_l^2\cos^2\varphi + c_t^2\sin^2\varphi}}.$$

The longitudinal group velocity U_{2l} and the transverse velocity U_{2t} are as follows:

$$U_{2l} = \sqrt{c_l^2\cos^2\varphi + c_t^2\sin^2\varphi}; \quad U_{2t} = \frac{c_l^2 - c_t^2}{\sqrt{c_l^2\cos^2\varphi - c_t^2\sin^2\varphi}}\sin\varphi\cdot\cos\varphi.$$

For a plane wave along x we have from (20)

$$\left(-\omega^2 + c_l^2k_x^2\right)u_x = 0; \quad \left(-\omega^2 + c_t^2k_x^2 + 2a^2\frac{k_x}{d}\right)u_y = 0.$$

Thus a longitudinal wave propagating along the magnetic field has the phase velocity c_l and the field has no effect on this wave. In the transverse wave propagating along **H** the magnetic field produces dispersion and the group velocity $U_t = (c_t^2\varkappa_x + a^2)/\sqrt{c_t^2\varkappa_x^2 + 2a^2\varkappa_x}$. For a plane wave propagating along the y axis perpendicular to **H** we get from (20) that

$$\left(-\omega^2 + c_l^2k_y^2 + 2a^2\frac{k_y}{d}\right)u_y = 0; \quad \left(-\omega^2 + c_t^2k_y^2\right)u_x = 0.$$

Here the longitudinal wave u_y has a dispersion similar to that of the transverse wave in the previous case with c_t replaced by c_l, and the transverse wave u_x propagates with a phase velocity c_t.

If the field H is large and the angles are close to $\pi/4$, we get from (15) that

$$-2a^2(k/d)\sin\alpha\cdot\cos\alpha u_x + (-\omega^2 + 2a^2(k/d)\cos^2\alpha)u_z = 0;$$
$$(-\omega^2 + 2a^2(k/d)\sin^2\alpha)u_x - 2a^2(k/d)\sin\alpha\cdot\cos\alpha\cdot u_z = 0;$$
$$(-\omega^2 + 2a^2k/d)u_y = 0.$$

The wave u_y propagates independently with a group velocity $(a/\sqrt{2\varkappa}\cdot\mathbf{k}/k)$. The dispersion relationship for the waves u_x, u_z

$$\omega^2 = 2a^2k/d$$

leads to a group velocity $a/\sqrt{2\varkappa}\cdot\mathbf{k}/k$. Thus all three deformations propagate along the vector **k** with an identical velocity U_H independently of the angle of inclination α.

We now give some numerical calculations. The Alfvén velocity is comparable with the velocity of sound in an elastic medium when $H^* \sim \sqrt{E}$ (for steel $E = 2\cdot10^6$ kg/cm², when $H^* \approx 10^6$ Oe). For a thin film $H^* \sim \sqrt{Ed/\lambda}$, where λ is the wavelength; i.e., when $d \ll \lambda$ the magnetic field begins to affect the deformation at much smaller values of H. Moreover, a conducting layer can be deposited on films of materials with small values of Young's modulus (for example, rubber, polyethylene, and so on), and for these the effects will begin to occur at magnetic fields of the order of a few oersteds.

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SCATTERING AND VELOCITY DISPERSION OF ULTRASONIC WAVES IN POLYCRYSTALS OF ORTHORHOMBIC SYMMETRY

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The scattering coefficients and the velocity of propagation of longitudinal and transverse ultrasonic waves in polycrystals of orthorhombic and higher symmetry are computed by the method of renormalization of the equations of motion. The formulas thus obtained are compared with the known asymptotic expressions for long and short waves. A numerical computation carried out for aluminum shows that for $qa \sim 1$ (q is the wave number; a is the correlation scale) the power index determining the frequency dependence of the scattering coefficient decreases monotonically from 4 to 2 for the transverse waves, while for the longitudinal waves this dependence is nonmonotonic, i.e., the power index decreases from 4 to 1, after which it increases again to 2. In the Rayleigh region ($q_l a < 1$) the scattering coefficient of the longitudinal waves increases with a power index smaller than 4.

A large number of studies has been devoted to the scattering of ultrasonic waves at the inhomogeneity grains of crystals; a review of these studies is given in [1]. The complexity of the computation leads to the re-

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